SURFACE WAVES IN AN ELASTIC HALF-SPACE IN AN ELECTRIC FIELD

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It was shown in several papers that the presence of electrostriction and the dependence of the dielectric constant ε on deformation lead to changes in the acoustic characteristics of dielectrics with a large dielectric constant, situated in an external electric field E_0 (see, for example, [1, 2]). In this paper, we show that an external electric field has an effect on surface waves. It is found that the rate of propagation of surface waves is appreciably influenced by the magnitude of the electric field and its orientation with respect to the surface of the medium and the direction of wave propagation. In sufficiently strong electric fields of a specific orientation, the occurrence of surface waves may prove to be altogether impossible.

The system which describes the coupled oscillations of the medium and the field is composed (for an isotropic elastic medium) of the following equations:

$$\frac{\partial}{\partial x_i}\{\varepsilon_0[\delta_{ik}-q_1\delta_{ik}\operatorname{div}u-q_2u_{ik}]E_k\}=0$$

$$\rho\ddot{u}_i=(\lambda+\mu)\nabla_i\operatorname{div}u+\mu\triangle u_i+\frac{\varepsilon_0}{4\pi}[q_1\nabla_iE^2+q_2(E\nabla)E_i+q_2E_i\operatorname{div}E] \tag{1}$$

where u_i is the displacement, u_{ik} is the strain tensor, ρ is the density, λ and μ are Lamé coefficients, $q_{1,2}$ are electrooptical coefficients, and $\varepsilon_0=\varepsilon$ for $u_{ik}=0$.

The boundary conditions for $Z=0$ have the form

$$\sigma_{ik}n_k=\sigma_{ik}^{\circ}n_k+\frac{\varepsilon_0}{4\pi}[q_1E^2\delta_{ik}+q_2E_iE_k]n_k=0 \tag{2}$$

where n_i is the unit vector of the normal to the surface, and $\overset{\circ}{\sigma}_{ik}$ is the stress tensor for $E_0=0$. The tangential electric-field component and the normal induction-vector component are continuous, while div $E=0$ beyond the region with $\varepsilon\gg1$.

By setting $u=u_l+u_t$, where the subscripts l and t denote quantities which refer to longitudinal and transverse waves, respectively, and taking into account that for surface waves all quantities are proportional (see, for example, [3]) to $\exp(\varkappa z-i\omega t+ikx)$ (for a wave propagating in direction x; the medium with $\varepsilon\gg1$ occupies a region $Z<0$), from (1) we obtain

$$\varkappa_l^2=k^2\frac{s_{0l}^2+s_{El}^2\cos^2\theta_k}{s_{0l}^2+s_{El}^2\cos^2\theta_\varkappa}-\frac{\omega^2}{s_{0l}^2+s_{El}^2\cos^2\theta_\varkappa} \tag{3}$$

$$\varkappa_t^2=k^2-\omega^2[s_{0t}^2+s_{Et}^2\cos^2\theta_u]^{-1} \tag{4}$$

$$s_{El}^2=\frac{\varepsilon_0E_0^2(q_1+q^2)^2}{4\pi\rho},\quad s_{Et}^2=\frac{\varepsilon_0E_0^2q_2^2}{8\pi\rho} \tag{5}$$

Here, s_{0lt} are the speeds of sound for $E_0=0$, and θ_k, θ_x, and θ_u denote the angles formed by direction E_0 and the axes X, Z, and direction u.

With the aid of (2)–(4) and $\operatorname{div}u_t=\operatorname{rot}u_l=0$, the dispersion equation for

$$\zeta=\frac{\omega}{k}[s_{0t}^2+s_{Et}^2\cos^2\theta_u]^{-1/2}$$

can be written in the form

$$\zeta^8-\zeta^6+8(3-2\alpha^2)\zeta^4-16\zeta^2(1-\alpha^2)+16(1-\beta^2)=0$$

$$\alpha^2=[s_{0t}^2+s_{Et}^2\cos^2\Theta_u][s_{0l}^2+s_{El}^2\cos^2\theta_\varkappa]^{-1}$$

$$\beta^2 = [s_{0l}^2 + s_{El}^2 \cos^2 \theta_k]\,[s_{0l}^2 + s_{El}^2 \cos^2 \theta_\varkappa]^{-1} \tag{6}$$

To the surface waves, there correspond the real roots of (6), which—as can be seen from (3) and (4)—must satisfy the double inequality

$$0 \leqslant \zeta^2 \leqslant \min\,[1,\ (\beta/\alpha)^2] \tag{7}$$

For $E_0 = 0$, inequality (6) reduces to the ordinary bicubic equation for ω/ks_{0t}.

It may be seen from (5) that for sufficiently strong electric fields, $s_{El,t} \gg s_{0l,t}$ can occur in media with $\varepsilon \gg 1$ (specific values of the fields and types of medium can be found in [2]). From here it follows that α^2, $\beta^2 \gg 1$, α^2, $\beta^2 \ll 1$ can occur for various magnitudes and orientations of the electric field. Let us analyze the solution of (6) for various α^2 and β^2.

α^2	$\beta^2 = 0.85$	0.90	1.0	α^2	$\beta^2 = 1.1$	1.2	1.3	1.5	2.0
0.0	0.36	0.38	0.985	1.1	0.9				
0.1		0.41	0.95	1.2	0.96				
0.2		0.43	0.94	1.4		0.67			
0.3		0.47	0.94	1.5	0.46	0.74	0.79		
0.4			0.93	2.0	0.33	0.47	0.56	0.77	
0.5			0.92	3.0	0.26	0.33	0.38	0.54	
0.6			0.915	5.0		0.04	0.18	0.29	0.64
0.7			0.89	10.0				0.04	0.36
0.8			0.79						
0.9			0.67						

The real roots of Eq. (6) that satisfy inequality (7) were obtained on a computer. It was found that such roots do not exist for all α^2 and β^2. For $\beta^2 < 0.85$, such roots do not exist. For $\beta^2 = 1$, Eq. (6) reduces to a bicubic equation, but as distinct from [3], α^2 varies almost down to zero for $E_0 \neq 0$. It can be seen from the table that for the same value of β^2, the roots need not depend monotonically on α^2. The roots exist always for $\beta^2 > 1$, provided $\alpha^2 > \beta^2$. Values of the roots for various α^2 and β^2 are tabulated.

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